Some Issues on Beam-Beam Interaction at CEPC

Y. Zhang
On behalf of CEPC Accelerator Study Group

Thanks: K. Ohmi, D. Shatilov, K. Oide, D. Zhou
Beam-beam parameter in e+/e- colliders


Machine Parameters of the KEKB  (June 17 2009)

<table>
<thead>
<tr>
<th></th>
<th>LER</th>
<th>HER</th>
</tr>
</thead>
<tbody>
<tr>
<td>C ircumference</td>
<td>3016 m</td>
<td>m</td>
</tr>
<tr>
<td>RF Frequency</td>
<td>508.88 MHz</td>
<td>MHz</td>
</tr>
<tr>
<td>Horizontal Emittance</td>
<td>18 nm</td>
<td>24 nm</td>
</tr>
<tr>
<td>Beam current</td>
<td>1637 mA</td>
<td>1188 mA</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>1584 + 1</td>
<td></td>
</tr>
<tr>
<td>Bunch current</td>
<td>1.03 mA</td>
<td>0.750 mA</td>
</tr>
<tr>
<td>Bunch spacing</td>
<td>1.84 m</td>
<td></td>
</tr>
<tr>
<td>Bunch trains</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Total RF volatage Vc</td>
<td>8.0 MV</td>
<td>13.0 MV</td>
</tr>
<tr>
<td>Synchrotron tune $V_s$</td>
<td>-0.0246</td>
<td>-0.0209</td>
</tr>
<tr>
<td>Betatron tune $V_x/V_y$</td>
<td>45.506/43.561</td>
<td>44.511/41.585</td>
</tr>
<tr>
<td>beta's at IP $\beta_x/\beta_y$</td>
<td>120/0.59</td>
<td>120/0.59 cm</td>
</tr>
<tr>
<td>momentum compaction $\alpha$</td>
<td>3.31 x 10^{-4}</td>
<td>3.43 x 10^{-4}</td>
</tr>
<tr>
<td>Estimated vertical beam size at IP from luminosity $\sigma_z^2$</td>
<td>0.94</td>
<td>0.94 $\mu$m</td>
</tr>
<tr>
<td>Beam-beam parameters $\xi_x/\xi_y$</td>
<td>0.127/0.129</td>
<td>0.102/0.090</td>
</tr>
<tr>
<td>Beam lifetime</td>
<td>133@1637 min.@mA</td>
<td>200@1188</td>
</tr>
<tr>
<td>Luminosity (Belle CsI)</td>
<td>21.08 $10^{33}$/cm²/sec</td>
<td></td>
</tr>
<tr>
<td>Luminosity records per day / 7 days / 30 days</td>
<td>1.479/8.428/30.208 /fb</td>
<td></td>
</tr>
</tbody>
</table>
Beam-Beam Parameter at CEPC & LEP2

http://tlep.web.cern.ch/content/accelerator-challenges

R. Assmann
Crab-Waist Compensation

Collision with large $\Phi$ is not a new idea ..... Crab-Waist transformation is!

$$y = \frac{x y'}{2 \theta}$$

**L_{geometric gain**

**x-y synchro-betatron and betatron resonance suppression**

\[ \Delta \nu_x = \frac{\pi}{2} \]

\[ \Delta \nu_y = \frac{\pi}{2} \]

sextupole    (anti)sextupole

$\beta_x, \beta_y$    $\beta_x^*, \beta_y^*$

P. Raimondi, *2nd SuperB Workshop, March 2006*

P. Raimondi, D. Shatilov, M. Zobov, physics/0702033

C. Milardi et al., Int. J. Mod. Phys. A24, 2009


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DAΦNE Luminosity and Tune Shift

<table>
<thead>
<tr>
<th></th>
<th>KLOE (Spt 2005)</th>
<th>FINUDA (Apr 2007)</th>
<th>SIDDHARTA CW (Jun 2009)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminosity [10^{32} cm^{-2}s^{-1}]</td>
<td>1.53</td>
<td>1.6</td>
<td>4.53 (5.0)</td>
</tr>
<tr>
<td>$l$(ele) [A]</td>
<td>1.38</td>
<td>1.50</td>
<td>1.52</td>
</tr>
<tr>
<td>$l$(pos) [A]</td>
<td>1.18</td>
<td>1.1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>111</td>
<td>106</td>
<td>105</td>
</tr>
<tr>
<td>$\alpha$ [mm mrad]</td>
<td>0.34</td>
<td>0.34</td>
<td>0.28</td>
</tr>
<tr>
<td>$\beta_x$ [m]</td>
<td>1.5</td>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>$\beta_y$ [cm]</td>
<td>1.8</td>
<td>1.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>0.0245</td>
<td>0.0291</td>
<td>0.0443 (0.074)</td>
</tr>
</tbody>
</table>

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Luminosity as a function of colliding currents

**CW-Sextupole excitation**

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L. CW SXT, OFF Feb 9th 2009

L. March 13th 2009
Estimation of Beamstrahlung lifetime

• Analysis

\[ \tau_{BS} \approx \frac{1}{n_{IP} f_{rev}} \frac{4\sqrt{\pi}}{3} \left( \frac{\delta_{acc}}{\alpha r_e} \right) \exp \left( \frac{2 \delta_{acc} \alpha}{3} \frac{\gamma \sigma_x \sigma_z}{r_e \gamma^2 \sqrt{2} r_e N_b} \right) \frac{\sqrt{2}}{\sqrt{\pi} \sigma_z \gamma^2 \left( \sqrt{2} r_e N_b \right)^{3/2}} \]


• Calculated by beam distribution

\[ \tau_{bs} = \frac{\tau_z}{2 A f(A)} \]

• \( A \) is the boundary of momentum acceptance in action,

• \( f(J) \) is the distribution of action with beam-beam, \( \int_0^\infty dJ f(J) = 1 \)

• \( \tau_z \) is the longitudinal damping time

K. Ohmi

![Graph showing lifetime vs. \( \eta \) with two calculation methods: Calc by Tail and Calc by Loss.](image)
Cross check of Beam-Beam Code

**Luminosity**

BBSS by K. Ohmi

**Bunch Length**

**Beamstrahlung Lifetime**

IBB by Y. Zhang
If the machine parameter is reasonable

- Limit of bunch population by beam-beam interaction
  - Beamstrahlung lifetime
  - If X-Z instability is suppressed
  - If asymmetric bunch current collision is stable
  - If there exist large enough stable working point space
  - If beam-beam parameter is safe enough
Tune Scan

\[ Q_y = 0.61, \quad Q_s = 0.035 \]

The error bar shows the turn-by-turn luminosity difference.

K. Ohmi and etal., DOI:10.1103/PhysRevLett.119.134801
\[ \xi_y = \frac{2r_e \beta_y^0 L}{N\gamma f_0} \]
Bunch Current Limit @ W

Graph 1:
- y-axis: \( \xi_{\text{lum}} \)
- x-axis: \( N_e \times 10^{10} \)
- Plot shows the relationship between \( N_e \) and \( \xi_{\text{lum}} \) with data points and a trend line.

Graph 2:
- x-axis: half turns
- y-axis: Luminosity/IP in \( 10^{34} \text{cm}^2\text{s}^{-1} \)
- Graphs for different \( n_e \) values (\( 15 \times 10^{10}, 16 \times 10^{10}, 17 \times 10^{10}, 18 \times 10^{10} \)) showing the luminosity over half turns.
Bunch Current Limit versus Horizontal Tune

Collision is stable in the range of [0.552, 0.555]

\[ 2v_x - 4v_s = n \]

\[ 2v_x - 6v_s = n \]

\[ N_p = 12 \times 10^{10} \]
Tune Scan @ W

Luminosity

$\sigma_x$

$\sigma_y$

Turn-by-Turn Luminosity Distortion
Tune Scan @ W (Qy=0.590)
Bootstrapping Injection

\[ N_p \propto \frac{\alpha \sigma \delta \sigma_2}{\beta_x^*} \] (K. Oide)

\[ \frac{\sigma_{x1}}{\sigma_{x0}} \]

\[ \frac{\sigma_{x2}}{\sigma_{x0}} \]

\[ N_D = 4.0 \cdot 10^{10} \]
\[ N_D = 4.0 \cdot 10^{10} \]
\[ N_D = 5.0 \cdot 10^{10} \]
\[ N_D = 5.0 \cdot 10^{10} \]
\[ N_D = 6.0 \cdot 10^{10} \]

\[ N_D = 4.0 \cdot 10^{10} \]
\[ N_D = 4.5 \cdot 10^{10} \]
\[ N_D = 4.5 \cdot 10^{10} \]
\[ N_D = 5.5 \cdot 10^{10} \]
\[ N_D = 5.5 \cdot 10^{10} \]
Bootstrapping is necessary? (15e10*15e10)
Bunch Current Limit versus Horizontal Tune

Criteria: $\frac{\sigma_x}{\sigma_0} < 1.1$

Width of safe $Q_x \sim 0.006$

- Collision is stable in the range of $[0.562, 0.568]$
- It is similar in case of $Z-2T$, with $n_p=12\times 10^{10}$

Z-3T
Bunch Current

Asymmetrical Bunch Current

Np=13e10 vs 15e10

Bunch Current Limit

Bunch Lengthening and Energy Spread w/o Collision
\[ \beta_y^* = 1.5\text{mm} \rightarrow 1\text{mm} \text{ (Solenoid: 3T} \rightarrow 2\text{T)} \]

With same beam current,

- smaller \( \beta_y^* \) + weaker solenoid, luminosity increase by a factor of two.
- bunch population increase from \( 8 \times 10^{10} \) to \( 12 \times 10^{10} \), luminosity increase about 20%.

\[ \xi_y = \frac{2\gamma r e \beta_y^0 L}{N_y f_0} \]
Crosstalk between Beam-Beam Interaction & Collective Effect

• Bunch length caused by impedance => $\sigma_z, \beta_z, \epsilon_z$

• In the conventional case, it is fine since the longitudinal dynamics is not sensitive to the beam-beam interaction

• In CEPC/FCC, the beamstrahlung effect will also lengthen the bunch

• It is self-consistent to consider the longitudinal wake field and beamstrahlung

\[
I(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \tilde{I}(\omega) \\
\tilde{V}(\omega) = -\tilde{I}(\omega)Z_\parallel(\omega)
\]
RMS size with longitudinal impedance
Beam-Beam Performance with longitudinal impedance
Dynamic Aperture & Lifetime: Case 0
Dynamic Aperture & Lifetime: Case 1
Dynamic Aperture & Lifetime: Case 2

- Plots showing the dynamic aperture for different parameters.
- Graphs illustrating the lifetime in minutes against various parameters.
- Visual representations of aperture size and lifetime trends.
Dynamic Aperture & Lifetime: Case 3
Dynamic Aperture & Lifetime: Case 4
Application of FMA to Beam-Beam Effects

Beam-beam resonances in the tune and amplitude planes for DAFNE, crab~0.4.

The diffusion index is calculated as $\log_{10}(\sigma_y)$, where $\sigma_y$ is the rms spread of tunes.
Could a So-Called Diffusion Map help us?

• FMA does not work well with chromaticity
• FMA will fail with strong synchrotron radiation and beamstrahlung effect
• In a lattice, with strong non-linearity, beam-beam interaction, strong SR fluctuation, could we construct a Diffusion Map to do some analysis?
Diffusion Process caused by Beam-Beam Interaction

• Diffusion Equation with a stochastic kick:
\[ \frac{\partial}{\partial s} \Psi(x, s) = B \frac{\partial^2}{\partial x^2} \Psi(x, s) \]

• For initial condition, \( \Psi(x, 0) = \delta(x - x_0) \), the solution is given as
\[ \Psi = \frac{1}{\sqrt{4\pi Bs}} \exp \left[ -\frac{(x - x_0)^2}{4Bs} \right] \]
which means \( \Psi \) is Gaussian and its rms value increase as \( \sigma^2 = 2Bs \)

• With damping \( \frac{dx}{ds} = -Dx \), the diffusion equation is replaced by Fokker-Plank equation
\[ \frac{\partial}{\partial s} \Psi(x, s) = Dx \frac{\partial}{\partial x} \Psi(x, s) + B \frac{\partial^2}{\partial x^2} \Psi(x, s) \]
and the equilibrium solution is \( \Psi = \exp \left[ -\frac{x^2}{2B/D} \right] \)

• In most of the circular accelerators, the betatron motion is much faster than the diffusion and damping, therefore the same discussion is applied by using the adiabatic invariant \( \sqrt{\mathcal{J}} \) instead of \( x \).

• If there are some diffusion mechanisms, which are independent each other, the total diffusion coefficient is summation of each diffusion coefficient, \( B = \sum B_i \)
MEASUREMENTS OF TRANSVERSE BEAM HALO DIFFUSION

The particle loss rate at the collimator is equal to the flux at that location:

\[ L = -D \left[ \partial_j f \right]_{j=J_C} \]

with phase-space density \( f(J, t) \) described by the diffusion equation, where \( J \) is the Hamiltonian action and \( D \) the diffusion coefficient in action space.

Figure 1: Schematic diagram of the apparatus (top). Example of the response of local loss rates to inward and outward collimator steps (bottom).
Figure 2.51: Left: The measured Poincaré map of the normalized phase-space coordinates \((x, p_x)\) of betatron motion near a third-order resonance \(3\nu_z = 11\) at the IUCF cooler ring. Note that particles outside the separatrix survive only about 100 turns. Tori for particles inside the separatrix are distorted by the third order resonance. The orientation of the Poincaré map, determined by sextupoles, rotates at a rate of betatron phase advance along the ring. The right plot shows the Poincaré map in action-angle variables \((J, \phi)\). The solid lines are Hamiltonian tori of Eq. (2.392).
Synchrotron Radiation


\[
\begin{pmatrix} \hat{X} \\ \hat{P} \end{pmatrix}_{out} = \lambda U(\mu) \begin{pmatrix} \hat{X} \\ \hat{P} \end{pmatrix}_{in} + \sqrt{\epsilon(1-\lambda^2)} \begin{pmatrix} \hat{r}_1 \\ \hat{r}_2 \end{pmatrix}.
\]

Let us calculate the ratio of radiation losses in quadrupoles and dipoles per unit FODO cell, which contains one QF and QD each. Using Eqs. (C3)-(C5),

\[
\frac{\langle P_Q \rangle}{\langle P_D \rangle} = \frac{\langle P_{QF} \rangle + \langle P_{QD} \rangle}{\langle P_D \rangle} = \frac{\ell_c^2}{2\theta_c^2} \left( \beta_{xQF}^2 \ell_{QF}^2 + \beta_{xQD}^2 \ell_{QD}^2 \right) n^2 \epsilon_x
\]

\[
\equiv R_Q n^2 \epsilon_x,
\]

where we have used \( n \equiv \Delta x/\sigma_x = \Delta x/\sqrt{\beta_{xQF}^2 \epsilon_x} \). Then the

FIG. 9. Poincaré plots in \( x = p_x \) (left) and \( z = \delta \) (right) planes for a particle starting at \( x = 10\sigma_x, \ p_x = y = p_y = z = \delta = 0 \), depicted by the red dots. The numbers, 0, 1, 2 are turns. The synchrotron radiation loss in the quadrupoles excites the synchrotron motion as shown in the right plot. The amount of the energy loss in the first turn \( \Delta p_1 \), and the peak amplitude of the synchrotron motion \( \Delta p \) agrees with the estimation, Eq. (C10).
Diffusion with Different Model

\[ A_i \overset{\text{def}}{=} \frac{2J_i}{\epsilon_i} \quad i = x, y, z \]

\[ \sigma_a \overset{\text{def}}{=} \sqrt{\sigma_{ax}^2 + \sigma_{ay}^2 + \sigma_{az}^2} \]

200 particles are tacked,
With same initial coordinate: \((3\sigma_x, 11\sigma_y)\)
Diffusion Map Analysis of Different Lattice

200 particles, 25 turns

\[ D \equiv \log_{10} \left( \sum_{\text{turn}} \sigma_d^2 \right) \]

The green lines show the boundary of \( D=1.6 \).

Slower diffusion indicates fewer halo particles.
Summary

• The beam-beam effect of CEPC CDR is briefly introduced
• The longitudinal impedance reduces the beam-beam limit according to the initial result.
• We attempt to present a diffusion map analysis method, which help to judge if one lattice is good in a non-symplectic condition. The initial result shows agreement with halo particles distribution obtained with many particle, long turns tracking.