Low-emittance tuning for circular colliders

Tessa Charles

With thanks to:
S. Aumon, B. Härer, B. Holzer, K. Oide, T. Tydecks, F. Zimmerman

eeFACT2018, Hong Kong
Challenges & constraints for FCC-ee (and CEPC) emittance tuning

\[ \beta_y^* = 1.6 \text{ mm} \]

\[ \beta_{x,\text{max}} = 1587.97 \text{ m} \]

\[ \beta_{y,\text{max}} = 6971.55 \text{ m} \]

Large emittance ratio, \( \epsilon_y / \epsilon_x = 0.201\% \)
Vertical dispersion & betatron coupling dominate $\varepsilon_y$ growth

Horizontal emittance:

$$\varepsilon_x = \frac{C_g}{J_x} \gamma^2 \theta^3 F$$

$$F_{FODO} = \frac{1}{2 \sin \psi} \frac{5 + 3 \cos \psi}{1 - \cos \psi} \frac{L}{l_B}$$

Vertical emittance:

$$\varepsilon_y = \left( \frac{dp}{p} \right)^2 \left( \gamma D_y^2 + 2 \alpha D_y D'_y + \beta D_y'^2 \right)$$

Sources of vertical emittance growth:

- vertical dispersion $D_y$
- betatron coupling
- opening angle $\sim \frac{1}{\gamma}$ (here negligible)

$L$: cell length
$l_B$: dipole length
$\phi$: phase advance/cell

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Correction methods used:

- **Orbit correction:**
  - MICADO & SVD from MADX
    - Hor. corrector at each QF, Vert. corrector at each QD
    - 1598 vertical correctors / 1590 horizontal correctors
    - BPM at each quadrupole
    - 1598 BPMs vertical / 1590 BPMs horizontal

- **Vertical dispersion and orbit:**
  - Orbit Dispersion Free Steering (DFS)
    $$\left( (1 - \alpha)\vec{u} \right) + \left( (1 - \alpha)\vec{A} \right)\vec{\theta} = 0$$

- **Linear coupling:**
  - Coupling resonant driving terms (RDT)
    - 1 skew at each sextupole + skews correctors at the IP
  
- **Beta beating correction & Horizontal dispersion via Response Matrix:**
  - Rematching of the phase advance at the BPMs
    - 1 trim quadrupole at each sextupole
    $$\left( \Delta\phi_{xy}, \Delta D_x \right) = R\Delta k_1$$

Until the end of 2009 coupling correction was performed over 32 parameters. The main drawbacks of this approach resulted in a nonlinear multidimensional minimization of the orbit. Adequate weight factors can be used to control the system to be solved, for example, to maintain a constant or-emittances on the corrector strengths being quadratic, exactly. Instead, an approximate solution must be found, for example, to maintain a constant or-emittance in the case of the ESRF storage ring the best correction is implemented in Ref. [34002-7]. A typical result before coupling correction is...
Initial assessment

**Quadrupoles:** $\Delta y = 2\ \mu m$

**Sextupoles:** $\Delta y = 10\ \mu m$

<table>
<thead>
<tr>
<th>Error type</th>
<th>$y,\ rms\ (mm)$</th>
<th>$D_{y,rms}\ (mm)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>quad arc ($\Delta y = 2\ \mu m$)</td>
<td>8.809</td>
<td>326.71</td>
</tr>
<tr>
<td>quad arc ($\Delta x = 10\ \mu m$)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>quad arc ($\Delta \phi = 10\ \mu rad$)</td>
<td>0.0</td>
<td>2.677</td>
</tr>
<tr>
<td>sextupoles ($\Delta y = 10\ \mu m$)</td>
<td>0.0245</td>
<td>57.13</td>
</tr>
<tr>
<td>sextupoles ($\Delta x = 10\ \mu m$)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>sextupoles ($\Delta \phi = 10\ \mu rad$)</td>
<td>0.0</td>
<td>0.004</td>
</tr>
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</table>
Correction methods applied to Vertical Quadrupole Misalignments ($\sigma_y = 100 \, \mu\text{m}$)

Sextupoles turned off:

Initial $D_y$
After orbit correction - factor 2e4 improvement
DFS - factor 50 improvement

Switch on sextupoles

Dispersion correction during the coupling correction

Factor $\sim 10$
Coupling matrix elements

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Beta-beating Correction – Weighted SVD

Beta-beat introduced through:

- Arc quads: $\Delta x = 100 \, \mu\text{m}$, $\Delta y = 100 \, \mu\text{m}$, $\Delta \theta = 100 \, \mu\text{m}$
- Sextupoles: $\Delta x = 100 \, \mu\text{m}$, $\Delta y = 100 \, \mu\text{m}$, $\Delta \theta = 100 \, \mu\text{m}$

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Correction Strategy

• Sextupoles strengths set to 0
  – x-y orbits correction
  – Coupling correction
  – Tune matching
  – Beat-beat correction
  – 1 step Dispersion Free Steering wo sextupole (Dy correction)
    + 1 step coupling correction (kicker strength change the coupling configuration)
  – Save x,x',y,y' at the beginning of the machine

• Set sextupoles strength to 10% of their design current
  – orbit corrections
  – coupling correction, tune matching
  – beta beat correction
  – coupling + Dy correction
  – increase by 10% the sextupole strength

• Final correction
  – Coupling corrections
  – Weighted beat-beat correction

7-8h up to one day of simulation/seed
Loop 20 times
This avoid the tunes run of to resonance and maximize the number of seeds

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Corrected Lattice
- Misaligned arc quads & sextupoles

<table>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
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IP quads perfectly aligned (for now)

436 out of 500 seeds converged

$\varepsilon_y = 0.093$ pm +/- 0.01
$\varepsilon_x = 1.520$ nm +/- 0.009
$\varepsilon_y / \varepsilon_x = 0.006\%$ (limit 0.1%)
Corrected Lattice
- Misaligned arc and IP quads & sextupoles

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700 out of 1000 seeds converged

$\varepsilon_y = 0.099$ pm +/- 0.013
$\varepsilon_x = 1.52$ nm +/- 0.01
$\varepsilon_y/\varepsilon_x = 0.0065\%$ (limit 0.1%)
Vertical dispersion highly susceptible misalignments

Dispersion introduced through: quadrupoles: $\Delta x = 100 \, \mu m$, $\Delta y = 100 \, \mu m$, $\Delta \theta = 100 \, \mu m$
Sextupoles: $\Delta x = 100 \, \mu m$, $\Delta y = 100 \, \mu m$, $\Delta \theta = 100 \, \mu m$

$D_{y,rms} = 261.1 \, m$

$D_{y,rms} = 1.02 \, mm$
Beta beat after correction
Corrected Lattice
- Misaligned arc and IP quads & sextupoles

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369 out of 1000 seeds converged

$\varepsilon_y = 0.11 \text{ pm } +/- 0.03$

$\varepsilon_x = 1.52 \text{ nm } +/- 0.01$

$\varepsilon_y/\varepsilon_x = 0.0073\%$ (limit 0.1%)
To increase the number of successful seeds:

- Add more tune matching to strategy.
- Start with relaxed optics, before reducing $\beta^*$.
- Place limit on maximum trim and skew quad strength that can be applied any given iteration step.
Dynamic / Momentum aperture
with radiation damping only

Apertures sufficient for beam storage and injection.

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Dynamic / Momentum aperture

with radiation damping and quantum excitation

Apertures sufficient for beam storage and injection.

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Conclusions

- FCC-ee poses a unique challenge for emittance tuning
- With 100 μm, 100 μrad misalignments in arc quads & sextupoles and 50μm and 50 μrad misalignments in IP quads, the mean vertical emittance achieved after correction schemes applied is $\varepsilon_y = 0.11$ pm rad
### Back up slides – top v213 lattice

The diagrams show the distribution of the variables $\varepsilon_x$, $\varepsilon_y$, and $\phi$ after correction. The histograms indicate the frequency of these variables across different ranges.

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Back up slides - DFS

• Build numerically a matrix for vertical orbit (u) & dispersion (D_u) response under a corrector kick (al)

\[
\left( \begin{array}{c} (1 - \alpha)u' \\ \alpha D_u' \end{array} \right) + \left( \begin{array}{c} (1 - \alpha)A \\ \alpha B \end{array} \right) \theta = 0
\]

• Orbit response

• Dispersion response

• SVD analysis to solve the system and find a solution

“Emittance optimization with dispersion free steering at LEP”
Back up slides - coupling

Coupling RDT \( f_{1001} - f_{1010} \) are related to the coupling parameter via:

\[
\Delta Q_{\text{min}} = |C^-| = \left| \frac{4\Delta}{2\pi R} \int ds f_{1001} e^{-i(\varphi_x - \varphi_y) + is\Delta/R} \right|,
\]

\( f_{1001} - f_{1010} \) can be computed via analytical formulas, or via a matrix formalism with the coupling matrix:

\[
f_{1001}^{1010} = \frac{\sum_W W_J w J_{w,1} \sqrt{\beta_x \beta_y} e^{i(\varphi_w x + \varphi_w y)}}{4(1 - e^{2\pi i(Q_x + Q_y)})}, \quad \Delta D_y = -(\Delta J_w) D_x \frac{\sqrt{\beta_y \beta_y 0}}{2\sin(\pi Q)} \cos(\pi Q - |\varphi_y 0 - \varphi_y|)
\]

A response matrix can be written to measure the response of the RDTs to a skew quadrupole field, \( J_c \). The system, which can be inverted via SVD:

\[
\begin{pmatrix}
\tilde{f}_{1001} \\
\tilde{f}_{1010}
\end{pmatrix}_{\text{meas}} = -M \tilde{J}_c,
\]

References:
- Vertical emittance reduction and preservation in electron storage rings via resonance driving terms correction, A. Franchi et al, PRSTAB 14, 034002
Back up slides – beta-beating

For $n$ trim quadrupoles which can exercise a small field strength $k_1$, the weighted SVD can be applied through adding weighting factors $f$ to each measurement of the beta-beat.

\[
\begin{pmatrix}
    f_1 \left( \frac{\beta_1 - \beta_{y0}}{\beta_{y0}} \right) \\
    f_2 \left( \frac{\beta_2 - \beta_{y0}}{\beta_{y0}} \right) \\
    \vdots \\
    f_m \left( \frac{\beta_m - \beta_{y0}}{\beta_{y0}} \right)
\end{pmatrix}_{\text{meas}} =
\begin{pmatrix}
    f_1 (R_{11}, R_{12}, R_{13}, \ldots, R_{1n}) \\
    f_2 (R_{21}, R_{22}, R_{23}, \ldots, R_{1n}) \\
    \vdots \\
    f_m (R_{m1}, R_{m2}, R_{m3}, \ldots, R_{mn})
\end{pmatrix} \times
\begin{pmatrix}
    k_1 \\
    k_2 \\
    \vdots \\
    k_n
\end{pmatrix}
\]

where $\beta_{y0}$ is the ideal beta function at the given BPM, $R_{i,j}$ are elements in the response matrix.