LOW-ENERGY ELECTRON-POSITRON COLLIDER TO SEARCH AND STUDY $(\mu^+\mu^-)$ BOUND STATE


BINP, Novosibirsk
Dimuonium

- Dimuonium, bimuonium or true muonium is a lepton atom ($\mu^+\mu^-$).
- From 6 leptonic atoms ($e^+e^-$), ($\mu^+e^-$), ($\mu^+\mu^-$), ($\tau^+e^-$), ($\tau^+\mu^-$), ($\tau^+\tau^-$) only two ($e^+e^-$), ($\mu^+e^-$) were observed.
- Dimuonium is pure QED system (no strong interaction, calculable).
- Very compact (large $m_\mu$), more sensitive to new physics than other exotic atoms.
Why dimuonium?

• Observation of the new classical QED object.
• QED test in the new regime.
• Experimental challenge leads to development of new techniques.
• Tests of muon properties are motivated by
  • $3.5 \sigma$ difference between $(\alpha-2)_\mu$ measurement and SM prediction,
  • discrepancies in the proton charge radius in muonic hydrogen,
  • hints of lepton-universality violation in rare B decays (LHCb), $B^+ \rightarrow K^+\ell^+\ell^-$ and $B^+ \rightarrow K^+\mu^+\mu^-$. 
Some references

• V.N.Baier and V.S.Synakh, Bimuonium production in electron-positron collisions, SOVIET PHYSICS JETP, 14, № 5, 1962, pp.1122-1125
  • Properties of the bound state, probability of observation

  • Very large crossing angle in order to eliminate background

• H. Lamm and R.F. Lebed, True Muonium ($\mu^+\mu^-$) on the Light Front, arXiv 1311.3245v3, 12 Nov 2014
  • Spectrum

• H. Lamm, True muonium: the atom that has it all, arXiv 1509.09306v1, 30 Sep 2016
  • Novel properties
Dimuonium properties

- Mass
  \[ M_{\mu\mu} = 2 \times 105.7 \text{ MeV} - 1.4 \text{ keV} \]
- Bohr radius
  \[ R_{\mu\mu} = 512 \text{ fm} \]
  \[ R_{ee} = 106000 \text{ fm} \]
- Muon lifetime 2.2 µs
- \(^3S_1\) states have photon quantum numbers \((J^{PC} = 1^{--})\); therefore could be produced in \(e^+e^-\) collisions

Dimuonium energy levels diagram

\[ n = \infty \text{ (} E = 0 \text{)} \]
\[ n = 3 \text{ (} E = -156 \text{ eV)} \]
\[ n = 2 \text{ (} E = -352 \text{ eV)} \]
\[ n = 1 \text{ (} E = -1407 \text{ eV)} \]
\[ 2 \text{ } P \]

\[ \gamma \text{ (15.4 ps)} \]
\[ \gamma \text{ (16.3 ps)} \]
\[ \gamma \text{ (4.81 ps)} \]
\[ \gamma \text{ (0.602 ps)} \]
\[ e^+e^- \text{ (14.5 ps)} \]
\[ e^+e^- \text{ (1.81 ps)} \]
Dimuonium production cross section

- Production of $n^3S_1$ in the $e^+e^- \rightarrow (\mu^+\mu^-) \rightarrow e^+e^-$

- $1^3S_1$: $\sigma(m_{\mu\mu}) \approx \frac{12\pi}{m_{\mu\mu}^2} \sqrt{\frac{\pi \Gamma_{ee}}{8 \sigma_M}} \approx 0.2 \frac{\Gamma_{ee}}{\sigma_M}$

  where $\sigma_M$ is center-of-mass energy spread

- For different collision schemes

  $\frac{\Gamma_{ee}}{\sigma_M} = \frac{0.37 \times 10^{-6} keV}{(7 \div 400) keV} \approx (1 \div 50) \times 10^{-9}$, $\sigma(m_{\mu\mu}) = 0.15 \div 7 \text{ nb}$

- Background: elastic $e^+e^- \rightarrow e^+e^-$ scattering

  - For crossing angle $45^\circ \div 135^\circ$ $\sigma_{Bhabha} = 22000 \text{ nb}$

  - Background/signal = $(5 \div 210) \times 10^3$

  - Background suppression is possible if decay point is separated from the origin point ($1^3S_1$: $ct = 540 \mu m$)
Head-on e+e- collision

\[ E_{\text{beam}} = 100 \div 150 \text{ MeV} \]
Collision monochromatization a la Reniery: 10 keV invariant mass resolution
\[ L \approx 10^{30} \text{ cm}^{-2}\text{s}^{-1} (\sim 50 (\mu^+\mu^-)/\text{hour}). \]

Observation of the dimuonium by searching for X-rays from (\(\mu^+\mu^-\)) Bohr transitions such as 2P\(\rightarrow\)1S (J.W.Moffat).

Failed due to large background.

Large crossing angle proposed by S.J.Brodsky and R.F.Lebed.

Large angle beam crossing

Invariant mass

\[
\langle M \rangle = 2E_0 \cos \theta - \frac{E_0}{2} \cos \theta \left[ \sigma_\delta^2 + \sigma_{px}^2 + \sigma_{py}^2 \frac{\cos 2\theta}{\cos \theta} \right]
\]

Invariant mass resolution

\[
\sigma_M^2 = 2E_0^2 \left[ \sigma_\delta^2 (\cos \theta)^2 + \sigma_{px}^2 (\sin \theta)^2 \right]
\]

Luminosity \((\varphi = \sigma_z \tan \theta / \sigma_x)\)

\[
L_0 = \frac{N_1 N_2}{4\pi \sigma_y \sigma_x \sqrt{1 + \varphi^2}} f_0 N_b \approx \frac{N_1 N_2}{4\pi \sigma_y \sigma_z \tan \theta} f_0 N_b
\]

Peak production rate

\[
\dot{N}_{\mu\mu} \approx \frac{\Gamma_{\mu\mu} \sigma_{\mu\mu} L_0}{2\sqrt{\pi} \sigma_M}
\]
Background

Decay length \((\mu^+\mu^- (1^3S_1) \rightarrow e^+e^-)\)

\[ OA = l = c \tau_{0,\mu\mu} \beta_{\mu\mu} \gamma_{\mu\mu} = c \tau_{0,\mu\mu} \tan \theta \]

Background: density of beam particles

\[ N_1 \propto \exp(-n_x^2/2) \]

\[ n_x = \frac{AB}{\sigma_x} = \frac{l \cos \theta}{\sigma_x} = c \tau_{0,\mu\mu} \sin \theta \]

Signal to background ratio

\[ \frac{\dot{N}_{\mu\mu}}{\dot{N}_{ee}} \propto \exp \left[ \frac{c^2 \tau_{0,\mu\mu}^2}{\sigma_x^2} \sin^2 \theta \right] \]

\[ \sqrt{\sigma_\delta^2 \cos^2 \theta + \left( \sigma_x^2 / \beta_x^2 \right) \sin^2 \theta} \]
Beam-beam effects with large crossing angle

Beam-beam tuneshift

\[ \xi_z = -\frac{N r_e}{2\pi\gamma} \frac{\alpha}{|\alpha|\sigma_\delta \sigma_z} \frac{\varphi^2}{1 + \varphi^2} \]

Hamiltonian

\[ \mathcal{H} = -\alpha \frac{p_z^2}{2} - \frac{v_s^2 z^2}{\alpha R^2} - \frac{2\xi_z v_s z^2}{\alpha R^2} \]

Population limit for \( \alpha > 0 \)

\[ N < \frac{2\pi R \gamma \alpha \sigma_\delta^2}{r_e} \]

\( \alpha < 0 \) has been studied at KEKB and at DAΦNE, no large currents, no luminosity due to microwave instability
Accelerator requirements

• Large positive momentum compaction (small circumference)
• Large crossing angle with small vertical beta function gives high luminosity (similar to crab waist)
• Large crossing angle 75° provides comfortable beam energy (e+ production) and decay length
  • beam energy $E_b = 408$ MeV
  • decay length $l(\mu^+\mu^- (1^3S_1)) = 2$ mm
• Higher signal to noise ratio requires $\sigma_x < c \tau_{0,\mu\mu} = 0.54$ mm
• Horizontal beam divergence contributes significant part in invariant mass resolution; therefore, low horizontal emittance
• Reverse of the beam direction provides 15° crossing angle and allows to study c.m. energy range from $\eta$ to $\eta'$ mesons (550-960 MeV)
Collider: overview
Collider: overview

Outer arc

Inner arc

Injection

RF cavities

IP1

IP2

Injection

RF cavities
Injection channels

0.7 m
Injection optics

- One turn injection in the horizontal plane
- K1 – K4 – running wave kickers
- Septum – Lambertson injection magnet
Injection trajectories

<table>
<thead>
<tr>
<th></th>
<th>L [cm]</th>
<th>H [kGs]</th>
<th>Fi [mrad]</th>
<th>U[kV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>50</td>
<td>0.057</td>
<td>2.1</td>
<td>± 22</td>
</tr>
<tr>
<td>K2</td>
<td>50</td>
<td>0.018</td>
<td>0.48</td>
<td>± 7</td>
</tr>
<tr>
<td>K3</td>
<td>50</td>
<td>0.057</td>
<td>2.1</td>
<td>± 22</td>
</tr>
<tr>
<td>K4</td>
<td>50</td>
<td>0.040</td>
<td>1.5</td>
<td>±16</td>
</tr>
<tr>
<td>SEPT2</td>
<td>40</td>
<td>3.5</td>
<td>100</td>
<td>--</td>
</tr>
</tbody>
</table>

Remaining oscillations ±10 $\sigma_x$
Dynamic aperture and energy acceptance

\[ \nu_x = 4.43 \]
\[ \nu_y = 3.37 \]
Dynamic aperture and energy acceptance

\[
\begin{align*}
  \nu_x &= 4.43 \\
  \nu_y &= 3.37
\end{align*}
\]
Interaction region

- Experimental chamber: flat box with 0.5-mm-thick beryllium windows on the top and on the bottom allowing passage of $e^\pm$ produced by the dimuonium atoms decay.
- Detector: tracking systems around the median plane, magnetic spectrometer
Interaction region

• **QD0**: permanent magnet, \( G = -35 \text{ T/m} \), \( \varnothing 30\text{mm} \)
• **QD/QF1**: electromagnet
### Collider: optics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>408 MeV</td>
</tr>
<tr>
<td>Circumference</td>
<td>29.35 m</td>
</tr>
<tr>
<td>Momentum compaction</td>
<td>$5.8 \times 10^{-2}$</td>
</tr>
<tr>
<td>Bunch intensity</td>
<td>$3.5 \times 10^{10}$ / 57 mA</td>
</tr>
<tr>
<td>Horizontal emittance</td>
<td>30 nm / 65 nm (IBS)</td>
</tr>
<tr>
<td>Energy spread</td>
<td>$3.7 \times 10^{-4}$ / $8.7 \times 10^{-4}$ (IBS)</td>
</tr>
<tr>
<td>$\beta_x / \beta_y$</td>
<td>150 mm / 2 mm</td>
</tr>
<tr>
<td>Luminosity</td>
<td>$2.8 \times 10^{30}$ cm$^{-2}$s$^{-1}$, Nb=1</td>
</tr>
<tr>
<td></td>
<td>$8.3 \times 10^{31}$ cm$^{-2}$s$^{-1}$, Nb=30</td>
</tr>
</tbody>
</table>
## Collider: parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF frequency</td>
<td>347.29 MHz</td>
</tr>
<tr>
<td>Beam energy</td>
<td>408.225 MeV</td>
</tr>
<tr>
<td>RF harmonic</td>
<td>34</td>
</tr>
<tr>
<td>Invariant mass (M)</td>
<td>211.315 MeV</td>
</tr>
<tr>
<td>RF voltage</td>
<td>510 kV</td>
</tr>
<tr>
<td>σ&lt;sub&gt;M&lt;/sub&gt;</td>
<td>383 keV</td>
</tr>
<tr>
<td>RF acceptance</td>
<td>2%</td>
</tr>
<tr>
<td>σ&lt;sub&gt;M&lt;/sub&gt;/M</td>
<td>1.8×10&lt;sup&gt;-3&lt;/sup&gt;</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>1.96×10&lt;sup&gt;-2&lt;/sup&gt;</td>
</tr>
<tr>
<td>IP beam divergence</td>
<td>6.4×10&lt;sup&gt;-4&lt;/sup&gt; (hor)</td>
</tr>
<tr>
<td>Damping partition</td>
<td>1.6 (hor) 1.4 (lon)</td>
</tr>
<tr>
<td>Energy spread</td>
<td>3.7×10&lt;sup&gt;-4&lt;/sup&gt; 8.7×10&lt;sup&gt;-4&lt;/sup&gt; (IBS)</td>
</tr>
<tr>
<td>Damping times</td>
<td>25 ms (hor) 40 ms (ver) 28 ms (lon)</td>
</tr>
<tr>
<td>Beam-beam tune shift</td>
<td>2×10&lt;sup&gt;-6&lt;/sup&gt; (hor) 1.1×10&lt;sup&gt;-3&lt;/sup&gt; (ver) -2×10&lt;sup&gt;-3&lt;/sup&gt; (lon)</td>
</tr>
<tr>
<td>Bunch length</td>
<td>5.1 mm 12 mm (IBS)</td>
</tr>
<tr>
<td>IP beam size at IP</td>
<td>97 μm (σ&lt;sub&gt;x&lt;/sub&gt;/IBS) 264 μm (σ&lt;sub&gt;x&lt;/sub&gt;/√2 cos θ)</td>
</tr>
</tbody>
</table>
Dimuonium production and distribution

- Detection efficiency is about 15% (2 IPs)
- $\beta\gamma c\tau = 2.03 \, mm$
- $\sigma_x(IP) = \sigma_x/(\sqrt{2}\cos \theta) = 280 \, \mu m$
- Detector vertex resolution is 150 $\mu m$
- Total $\sigma_{vtx} = 320 \, \mu m$
- 6.25 $\sigma$ background suppression with vertex position $x > 2 \, mm$

<table>
<thead>
<tr>
<th>$\mu^+\mu^-$ rate</th>
<th>1 hour</th>
<th>4 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &gt; 2 , mm$</td>
<td>1S/2S/3S</td>
<td>4.7/1.4/0.5</td>
</tr>
</tbody>
</table>

![Graphs showing dimuonium production and distribution](image)
Experiments: what can we measure?

- From the fit of the decay vertex distribution
  - dimuonium production rate ($\Gamma_{ee}$) of 1S (2% for $10^7$ s), 2S(15%), 3S(30%)
  - dimuonium decay lengths with the same accuracy
- Dimuonium interaction with a thin foil (30µm Al) allows
  - measurement of the breakup probability
  - measurement 1S-2P transition probabilities
  - 2P lifetime
- Laser spectroscopy (?)
  - $\Delta E(2S-2P)$ (laser $\lambda \approx 100\mu m$)
  - 2P lifetime
Experiments: $e^+ e^- \rightarrow \mu^+ \mu^-$ near threshold

Coulomb interaction in the final state leads to nonzero cross section at the threshold; therefore,

- Background-free measurement of the cross section near the threshold, requires magnetic spectrometer
- Precision measurement of the SSSG-factor
- C.M. energy and its spread calibration
- The same technique may be used for $e^+ e^- \rightarrow \pi^+ \pi^-$

Born cross section + rad. correction + energy spread with $\delta=400$ keV
Experiments: 15° crossing angle

• This region (c.m. 550-960 MeV) of $\rho$ and $\omega$ resonances is important for SM $(g-2)_\mu$ calculation

• $e^+e^- \rightarrow \pi^+\pi^-$ cross section measurement with unlimited statistics

• Precision measurements of other hadronic cross sections
  
  $(e^+e^- \rightarrow \pi^+\pi^-\pi^0, \pi^0\gamma, \eta\gamma, \pi^0\pi^0\gamma, 4\pi, \cdots)$

• Rare processes $e^+e^- \rightarrow \eta, \eta'$

• Two-photon processes $\gamma\gamma \rightarrow \pi^0, \pi\pi, \eta$

• Measurement of meson-photon transition form factors
Reverse beam: 15° crossing angle

<table>
<thead>
<tr>
<th></th>
<th>Beam energy</th>
<th>$\eta$</th>
<th>$\eta'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invariant mass (M)</td>
<td>547.86 MeV</td>
<td>957.76 MeV</td>
<td></td>
</tr>
<tr>
<td>$\sigma_M$ ($\sigma_M/M$)</td>
<td>420 keV (7.7×10^{-4})</td>
<td>580 keV (6.1×10^{-4})</td>
<td></td>
</tr>
<tr>
<td>Energy spread</td>
<td>$2.8\times10^{-4}$/10.6×10^{-4} (IBS)</td>
<td>$4.8\times10^{-4}$/8.4×10^{-4} (IBS)</td>
<td></td>
</tr>
<tr>
<td>IP beam divergence (hor)</td>
<td>8.3×10^{-4}</td>
<td>7.1×10^{-4}</td>
<td></td>
</tr>
<tr>
<td>Horizontal emittance</td>
<td>11.4 nm / 105 nm (IBS)</td>
<td>34.8 nm / 75 nm (IBS)</td>
<td></td>
</tr>
<tr>
<td>Bunch length</td>
<td>3.7 mm / 14.2 mm (IBS)</td>
<td>6.3 mm / 11 mm (IBS)</td>
<td></td>
</tr>
<tr>
<td>Beam-beam $\zeta$ (h/v/l)</td>
<td>$3\times10^{-4}$/1.4×10^{-2}/-2×10^{-3}</td>
<td>$3\times10^{-4}$/1.3×10^{-2}/-2×10^{-3}</td>
<td></td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>1.67×10^{-2}</td>
<td>1.71×10^{-2}</td>
<td></td>
</tr>
<tr>
<td>Luminosity (Nb=1 / 30)</td>
<td>3.3×10^{31} / 9.9×10^{32}</td>
<td>5.2×10^{31} / 1.5×10^{33}</td>
<td></td>
</tr>
</tbody>
</table>
Conclusion

• Collider to observe and study bound state of \((\mu^+\mu^-)\)
  • two rings, large crossing angle
  • circumference 29 m, not expensive to build and operate
  • luminosity \(8 \times 10^{31} \text{ cm}^{-2}\text{s}^{-1}\)
• Reverse of the beam allows experiments in 500-1000 MeV central mass energy range
• Geometry is optimized to fit the present hall.
• Dynamic aperture and energy acceptance are sufficient for injection and beam life time.
• Details are in https://doi.org/10.1051/epjconf/201818101032
• We started manufacturing of the collider components.

We are open for collaboration and experiments proposals